

1. Table: Clebsch-Gordan coefficients, spherical harmonics, and d functions⁵

Sign convention is that of E. P. Wigner, *Group Theory* (Academic, New York, 1959), also used by E. V. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University, New York, 1953), M. E. Rose, *Elementary Theory of Angular Momentum* (Wiley, New York, 1957), and Cohen, *Tables of the Clebsch-Gordan Coefficients* (North American Rockwell Science Center, Thousand Oaks, CA, 1974). The signs and numbers in the current tables have been calculated by computer programs written independently by Cohen and at LBL.

Note: A $\sqrt{\quad}$ is to be understood over every coefficient; e.g., for $-8/15$ read $-\sqrt{8/15}$.

Notation: $\begin{matrix} J & J & \dots \\ M & M & \dots \end{matrix}$

$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta$

$Y_1^1 = -\sqrt{\frac{3}{8\pi}} \sin\theta e^{i\phi}$

$Y_2^0 = \sqrt{\frac{5}{4\pi}} \left(\frac{3}{2} \cos^2\theta - \frac{1}{2} \right)$

$Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta e^{i\phi}$

$Y_2^2 = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2\theta e^{2i\phi}$

$d_{m,0}^J = \sqrt{\frac{4\pi}{2J+1}} Y_J^{m,0} e^{-im\phi}$

$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M \rangle = (-1)^{J-j_1-j_2} \langle j_2 j_1 m_2 m_1 | j_2 j_1 J M \rangle$

$d_{m',m}^j = (-1)^{m-m'} d_{m,m'}^j = d_{-m,-m'}^j$

$d_{1/2,1/2}^{1/2} = \cos\theta$ $d_{1/2,-1/2}^{1/2} = -\sin\theta$

$d_{1,1}^1 = \frac{1+\cos\theta}{2}$ $d_{1,0}^1 = -\frac{\sin\theta}{\sqrt{2}}$

$d_{1,-1}^1 = \frac{1-\cos\theta}{2}$ $d_{0,0}^1 = \cos\theta$

$d_{3/2,3/2}^{3/2} = \frac{1+\cos\theta}{2} \cos\theta$

$d_{3/2,1/2}^{3/2} = -\sqrt{3} \frac{1+\cos\theta}{2} \sin\theta$

$d_{3/2,-1/2}^{3/2} = \sqrt{3} \frac{1-\cos\theta}{2} \cos\theta$

$d_{3/2,-3/2}^{3/2} = -\frac{1-\cos\theta}{2} \sin\theta$

$d_{2,2}^2 = \left(\frac{1+\cos\theta}{2} \right)^2$

$d_{2,1}^2 = -\frac{1+\cos\theta}{2} \sin\theta$

$d_{2,0}^2 = \frac{\sqrt{6}}{4} \sin^2\theta$

$d_{2,-1}^2 = -\frac{1-\cos\theta}{2} \sin\theta$

$d_{2,-2}^2 = \left(\frac{1-\cos\theta}{2} \right)^2$

$d_{1,1}^2 = \frac{1+\cos\theta}{2} (2\cos\theta - 1)$

$d_{1,0}^2 = -\sqrt{\frac{3}{2}} \sin\theta \cos\theta$

$d_{1,-1}^2 = \frac{1-\cos\theta}{2} (2\cos\theta + 1)$

$d_{0,0}^2 = \left(\frac{3}{2} \cos^2\theta - \frac{1}{2} \right)$

Tables of Clebsch-Gordan coefficients and d functions for various angular momentum values (J, M) and magnetic quantum numbers (m1, m2).