Exact Values of Trigonometric Functions at 3° Intervals

David G. Simpson, Ph.D.

August 11, 2014

The following tables give the exact values of trigonometric functions of angles at 3° intervals. (Ref.: E. Gelin, *Éléments de Trigonométrie plane et sphérique à l'usage des élèves des Cours professionnels des candidats aux Écoles spéciales des Universités et à l'École militaire de Bruxelles* (1888).)

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$0° = 0\pi$	0	1	0
$3° = \frac{\pi}{60}$	$\frac{1}{16}\left[(\sqrt{6}+\sqrt{2})(\sqrt{5}-1)-2(\sqrt{3}-1)\sqrt{5+\sqrt{5}}\right]$	$\frac{1}{16}\left[2(\sqrt{3}+1)\sqrt{5+\sqrt{5}}+(\sqrt{6}-\sqrt{2})(\sqrt{5}-1)\right]$	$\frac{1}{4}(\sqrt{5}-\sqrt{3})(\sqrt{3}-1)\left(\sqrt{10+2\sqrt{5}}-\sqrt{5}-1\right)$
$6° = \frac{\pi}{30}$	$\frac{1}{8}\left(\sqrt{30-6\sqrt{5}}-\sqrt{5}-1\right)$	$\frac{1}{8}\left(\sqrt{15}+\sqrt{3}+\sqrt{10-2\sqrt{5}}\right)$	$\frac{1}{2}\left(\sqrt{10-2\sqrt{5}}-\sqrt{15}+\sqrt{3}\right)$
$9° = \frac{\pi}{20}$	$\frac{1}{8}\left(\sqrt{10}+\sqrt{2}-2\sqrt{5-\sqrt{5}}\right)$	$\frac{1}{8}\left(\sqrt{10}+\sqrt{2}+2\sqrt{5-\sqrt{5}}\right)$	$\sqrt{5}+1-\sqrt{5+2\sqrt{5}}$
$12° = \frac{\pi}{15}$	$\frac{1}{8}\left(\sqrt{10+2\sqrt{5}}-\sqrt{15}+\sqrt{3}\right)$	$\frac{1}{8}\left(\sqrt{30+6\sqrt{5}}+\sqrt{5}-1\right)$	$\frac{1}{2}\left(3\sqrt{3}-\sqrt{15}-\sqrt{50-22\sqrt{5}}\right)$
$15° = \frac{\pi}{12}$	$\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$2-\sqrt{3}$
$18° = \frac{\pi}{10}$	$\frac{1}{4}(\sqrt{5}-1)$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\frac{1}{5}\sqrt{25-10\sqrt{5}}$
$21° = \frac{7\pi}{60}$	$\frac{1}{16}\left[2(\sqrt{3}+1)\sqrt{5-\sqrt{5}}-(\sqrt{6}-\sqrt{2})(\sqrt{5}+1)\right]$	$\frac{1}{16}\left[(\sqrt{6}+\sqrt{2})(\sqrt{5}+1)+2(\sqrt{3}-1)\sqrt{5-\sqrt{5}}\right]$	$\frac{1}{4}(\sqrt{5}-\sqrt{3})(\sqrt{3}+1)\left(\sqrt{10-2\sqrt{5}}-\sqrt{5}+1\right)$
$24° = \frac{2\pi}{15}$	$\frac{1}{8}\left(\sqrt{15}+\sqrt{3}-\sqrt{10-2\sqrt{5}}\right)$	$\frac{1}{8}\left(\sqrt{30-6\sqrt{5}}+\sqrt{5}+1\right)$	$\frac{1}{2}\left(\sqrt{50+22\sqrt{5}}-3\sqrt{3}-\sqrt{15}\right)$
$27° = \frac{3\pi}{20}$	$\frac{1}{8}\left(2\sqrt{5+\sqrt{5}}-\sqrt{10}+\sqrt{2}\right)$	$\frac{1}{8}\left(2\sqrt{5+\sqrt{5}}+\sqrt{10}-\sqrt{2}\right)$	$\sqrt{5}-1-\sqrt{5-2\sqrt{5}}$
$30° = \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{3}\sqrt{3}$
$33° = \frac{11\pi}{60}$	$\frac{1}{16}\left[(\sqrt{6}+\sqrt{2})(\sqrt{5}-1)+2(\sqrt{3}-1)\sqrt{5+\sqrt{5}}\right]$	$\frac{1}{16}\left[2(\sqrt{3}+1)\sqrt{5+\sqrt{5}}-(\sqrt{6}-\sqrt{2})(\sqrt{5}-1)\right]$	$\frac{1}{4}(\sqrt{5}-\sqrt{3})(\sqrt{3}-1)\left(\sqrt{10+2\sqrt{5}}+\sqrt{5}+1\right)$
$36° = \frac{\pi}{5}$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$\frac{1}{4}(\sqrt{5}+1)$	$\sqrt{5-2\sqrt{5}}$
$39° = \frac{13\pi}{60}$	$\frac{1}{16}\left[(\sqrt{6}+\sqrt{2})(\sqrt{5}+1)-2(\sqrt{3}-1)\sqrt{5-\sqrt{5}}\right]$	$\frac{1}{16}\left[2(\sqrt{3}+1)\sqrt{5-\sqrt{5}}+(\sqrt{6}-\sqrt{2})(\sqrt{5}+1)\right]$	$\frac{1}{4}(\sqrt{5}+\sqrt{3})(\sqrt{3}-1)\left(\sqrt{10-2\sqrt{5}}-\sqrt{5}+1\right)$
$42° = \frac{7\pi}{30}$	$\frac{1}{8}\left(\sqrt{30+6\sqrt{5}}-\sqrt{5}+1\right)$	$\frac{1}{8}\left(\sqrt{10+2\sqrt{5}}+\sqrt{15}-\sqrt{3}\right)$	$\frac{1}{2}\left(\sqrt{15}+\sqrt{3}-\sqrt{10+2\sqrt{5}}\right)$
$45° = \frac{\pi}{4}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$	1
$48° = \frac{4\pi}{15}$	$\frac{1}{8}\left(\sqrt{10+2\sqrt{5}}+\sqrt{15}-\sqrt{3}\right)$	$\frac{1}{8}\left(\sqrt{30+6\sqrt{5}}-\sqrt{5}+1\right)$	$\frac{1}{2}\left(3\sqrt{3}-\sqrt{15}+\sqrt{50-22\sqrt{5}}\right)$
$51° = \frac{17\pi}{60}$	$\frac{1}{16}\left[2(\sqrt{3}+1)\sqrt{5-\sqrt{5}}+(\sqrt{6}-\sqrt{2})(\sqrt{5}+1)\right]$	$\frac{1}{16}\left[(\sqrt{6}+\sqrt{2})(\sqrt{5}+1)-2(\sqrt{3}-1)\sqrt{5-\sqrt{5}}\right]$	$\frac{1}{4}(\sqrt{5}-\sqrt{3})(\sqrt{3}+1)\left(\sqrt{10-2\sqrt{5}}+\sqrt{5}-1\right)$
$54° = \frac{3\pi}{10}$	$\frac{1}{4}(\sqrt{5}+1)$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$\frac{1}{5}\sqrt{25+10\sqrt{5}}$
$57° = \frac{19\pi}{60}$	$\frac{1}{16}\left[2(\sqrt{3}+1)\sqrt{5+\sqrt{5}}-(\sqrt{6}-\sqrt{2})(\sqrt{5}-1)\right]$	$\frac{1}{16}\left[(\sqrt{6}+\sqrt{2})(\sqrt{5}-1)+2(\sqrt{3}-1)\sqrt{5+\sqrt{5}}\right]$	$\frac{1}{4}(\sqrt{5}+\sqrt{3})(\sqrt{3}+1)\left(\sqrt{10+2\sqrt{5}}-\sqrt{5}-1\right)$
$60° = \frac{\pi}{3}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}$	$\sqrt{3}$
$63° = \frac{7\pi}{20}$	$\frac{1}{8}\left(2\sqrt{5+\sqrt{5}}+\sqrt{10}-\sqrt{2}\right)$	$\frac{1}{8}\left(2\sqrt{5+\sqrt{5}}-\sqrt{10}+\sqrt{2}\right)$	$\sqrt{5}-1+\sqrt{5-2\sqrt{5}}$
$66° = \frac{11\pi}{30}$	$\frac{1}{8}\left(\sqrt{30-6\sqrt{5}}+\sqrt{5}+1\right)$	$\frac{1}{8}\left(\sqrt{15}+\sqrt{3}-\sqrt{10-2\sqrt{5}}\right)$	$\frac{1}{2}\left(\sqrt{10-2\sqrt{5}}+\sqrt{15}-\sqrt{3}\right)$
$69° = \frac{23}{60}\pi$	$\frac{1}{16}\left[(\sqrt{6}+\sqrt{2})(\sqrt{5}+1)+2(\sqrt{3}-1)\sqrt{5-\sqrt{5}}\right]$	$\frac{1}{16}\left[2(\sqrt{3}+1)\sqrt{5-\sqrt{5}}-(\sqrt{6}-\sqrt{2})(\sqrt{5}+1)\right]$	$\frac{1}{4}(\sqrt{5}+\sqrt{3})(\sqrt{3}-1)\left(\sqrt{10-2\sqrt{5}}+\sqrt{5}-1\right)$
$72° = \frac{2\pi}{5}$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\frac{1}{4}(\sqrt{5}-1)$	$\sqrt{5+2\sqrt{5}}$
$75° = \frac{5\pi}{12}$	$\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$2+\sqrt{3}$
$78° = \frac{13\pi}{30}$	$\frac{1}{8}\left(\sqrt{30+6\sqrt{5}}+\sqrt{5}-1\right)$	$\frac{1}{8}\left(\sqrt{10+2\sqrt{5}}-\sqrt{15}+\sqrt{3}\right)$	$\frac{1}{2}\left(\sqrt{15}+\sqrt{3}+\sqrt{10+2\sqrt{5}}\right)$
$81° = \frac{19\pi}{20}$	$\frac{1}{8}\left(\sqrt{10}+\sqrt{2}+2\sqrt{5-\sqrt{5}}\right)$	$\frac{1}{8}\left(\sqrt{10}+\sqrt{2}-2\sqrt{5-\sqrt{5}}\right)$	$\sqrt{5}+1+\sqrt{5+2\sqrt{5}}$
$84° = \frac{7\pi}{15}$	$\frac{1}{8}\left(\sqrt{15}+\sqrt{3}+\sqrt{10-2\sqrt{5}}\right)$	$\frac{1}{8}\left(\sqrt{30-6\sqrt{5}}-\sqrt{5}-1\right)$	$\frac{1}{2}\left(\sqrt{50+22\sqrt{5}}+3\sqrt{3}+\sqrt{15}\right)$
$87° = \frac{29\pi}{60}$	$\frac{1}{16}\left[2(\sqrt{3}+1)\sqrt{5+\sqrt{5}}+(\sqrt{6}-\sqrt{2})(\sqrt{5}-1)\right]$	$\frac{1}{16}\left[(\sqrt{6}+\sqrt{2})(\sqrt{5}-1)-2(\sqrt{3}-1)\sqrt{5+\sqrt{5}}\right]$	$\frac{1}{4}(\sqrt{5}+\sqrt{3})(\sqrt{3}+1)\left(\sqrt{10+2\sqrt{5}}+\sqrt{5}+1\right)$
$90° = \frac{\pi}{2}$	1	0	∞

θ	$\sec\theta$	$\csc\theta$	$\cot\theta$
$0^\circ = 0\pi$	1	∞	∞
$3^\circ = \frac{\pi}{60}$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(\sqrt{5+2\sqrt{5}}-2+\sqrt{3}\right)$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(2+\sqrt{3}+\sqrt{5+2\sqrt{5}}\right)$	$\frac{1}{4}\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{3}+1\right)\left(\sqrt{10+2\sqrt{5}}+\sqrt{5}+1\right)$
$6^\circ = \frac{\pi}{30}$	$\sqrt{3}-\sqrt{5-2\sqrt{5}}$	$\sqrt{15+6\sqrt{5}}+\sqrt{5}+2$	$\frac{1}{2}\left(\sqrt{50+22\sqrt{5}}+3\sqrt{3}+\sqrt{15}\right)$
$9^\circ = \frac{\pi}{20}$	$\frac{1}{2}\left(3\sqrt{2}+\sqrt{10}-2\sqrt{5+\sqrt{5}}\right)$	$\frac{1}{2}\left(3\sqrt{2}+\sqrt{10}+2\sqrt{5}+\sqrt{5}\right)$	$\sqrt{5}+1+\sqrt{5+2\sqrt{5}}$
$12^\circ = \frac{\pi}{15}$	$\sqrt{15-6\sqrt{5}}-\sqrt{5}+2$	$\sqrt{5+2\sqrt{5}}+\sqrt{3}$	$\frac{1}{2}\left(\sqrt{15}+\sqrt{3}+\sqrt{10+2\sqrt{5}}\right)$
$15^\circ = \frac{\pi}{12}$	$\sqrt{6}-\sqrt{2}$	$\sqrt{6}+\sqrt{2}$	$2+\sqrt{3}$
$18^\circ = \frac{\pi}{10}$	$\frac{1}{5}\sqrt{50-10\sqrt{5}}$	$\sqrt{5}+1$	$\sqrt{5+2\sqrt{5}}$
$21^\circ = \frac{7\pi}{60}$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(2+\sqrt{3}-\sqrt{5-2\sqrt{5}}\right)$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(\sqrt{5-2\sqrt{5}}+2-\sqrt{3}\right)$	$\frac{1}{4}\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{3}-1\right)\left(\sqrt{10-2\sqrt{5}}+\sqrt{5}-1\right)$
$24^\circ = \frac{2\pi}{15}$	$\sqrt{15+6\sqrt{5}}-\sqrt{5}-2$	$\sqrt{3}+\sqrt{5-2\sqrt{5}}$	$\frac{1}{2}\left(\sqrt{10-2\sqrt{5}}+\sqrt{15}-\sqrt{3}\right)$
$27^\circ = \frac{3\pi}{20}$	$\frac{1}{2}\left(2\sqrt{5-\sqrt{5}}-3\sqrt{2}+\sqrt{10}\right)$	$\frac{1}{2}\left(2\sqrt{5-\sqrt{5}}+3\sqrt{2}-\sqrt{10}\right)$	$\sqrt{5}-1+\sqrt{5-2\sqrt{5}}$
$30^\circ = \frac{\pi}{6}$	$\frac{2}{3}\sqrt{3}$	2	$\sqrt{3}$
$33^\circ = \frac{11\pi}{60}$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(\sqrt{5+2\sqrt{5}}+2-\sqrt{3}\right)$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(2+\sqrt{3}-\sqrt{5+2\sqrt{5}}\right)$	$\frac{1}{4}\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{3}+1\right)\left(\sqrt{10+2\sqrt{5}}-\sqrt{5}-1\right)$
$36^\circ = \frac{\pi}{5}$	$\sqrt{5}-1$	$\frac{1}{5}\sqrt{50+10\sqrt{5}}$	$\frac{1}{5}\sqrt{25+10\sqrt{5}}$
$39^\circ = \frac{13\pi}{60}$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(\sqrt{5-2\sqrt{5}}-2+\sqrt{3}\right)$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(2+\sqrt{3}+\sqrt{5-2\sqrt{5}}\right)$	$\frac{1}{4}\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{3}+1\right)\left(\sqrt{10-2\sqrt{5}}+\sqrt{5}-1\right)$
$42^\circ = \frac{7\pi}{30}$	$\sqrt{5+2\sqrt{5}}-\sqrt{3}$	$\sqrt{15-6\sqrt{5}}+\sqrt{5}-2$	$\frac{1}{2}\left(3\sqrt{3}-\sqrt{15}+\sqrt{50-22\sqrt{5}}\right)$
$45^\circ = \frac{\pi}{4}$	$\sqrt{2}$	$\sqrt{2}$	1
$48^\circ = \frac{4\pi}{15}$	$\sqrt{15-6\sqrt{5}}+\sqrt{5}-2$	$\sqrt{5+2\sqrt{5}}-\sqrt{3}$	$\frac{1}{2}\left(\sqrt{15}+\sqrt{3}-\sqrt{10+2\sqrt{5}}\right)$
$51^\circ = \frac{17\pi}{60}$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(2+\sqrt{3}+\sqrt{5-2\sqrt{5}}\right)$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(\sqrt{5-2\sqrt{5}}-2+\sqrt{3}\right)$	$\frac{1}{4}\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{3}-1\right)\left(\sqrt{10-2\sqrt{5}}-\sqrt{5}+1\right)$
$54^\circ = \frac{3\pi}{10}$	$\frac{1}{5}\sqrt{50+10\sqrt{5}}$	$\sqrt{5}-1$	$\sqrt{5-2\sqrt{5}}$
$57^\circ = \frac{19\pi}{60}$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(2+\sqrt{3}-\sqrt{5+2\sqrt{5}}\right)$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(\sqrt{5+2\sqrt{5}}+2-\sqrt{3}\right)$	$\frac{1}{4}\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{3}-1\right)\left(\sqrt{10+2\sqrt{5}}+\sqrt{5}+1\right)$
$60^\circ = \frac{\pi}{3}$	2	$\frac{2}{3}\sqrt{3}$	$\frac{1}{3}\sqrt{3}$
$63^\circ = \frac{7\pi}{20}$	$\frac{1}{2}\left(2\sqrt{5-\sqrt{5}}+3\sqrt{2}-\sqrt{10}\right)$	$\frac{1}{2}\left(2\sqrt{5-\sqrt{5}}-3\sqrt{2}+\sqrt{10}\right)$	$\sqrt{5}-1-\sqrt{5-2\sqrt{5}}$
$66^\circ = \frac{11\pi}{30}$	$\sqrt{3}+\sqrt{5-2\sqrt{5}}$	$\sqrt{15+6\sqrt{5}}-\sqrt{5}-2$	$\frac{1}{2}\left(\sqrt{50+22\sqrt{5}}-3\sqrt{3}-\sqrt{15}\right)$
$69^\circ = \frac{23}{60}\pi$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(\sqrt{5-2\sqrt{5}}+2-\sqrt{3}\right)$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(2+\sqrt{3}-\sqrt{5-2\sqrt{5}}\right)$	$\frac{1}{4}\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{3}+1\right)\left(\sqrt{10-2\sqrt{5}}-\sqrt{5}+1\right)$
$72^\circ = \frac{2\pi}{5}$	$\sqrt{5}+1$	$\frac{1}{5}\sqrt{50-10\sqrt{5}}$	$\frac{1}{5}\sqrt{25-10\sqrt{5}}$
$75^\circ = \frac{5\pi}{12}$	$\sqrt{6}+\sqrt{2}$	$\sqrt{6}-\sqrt{2}$	$2-\sqrt{3}$
$78^\circ = \frac{13\pi}{30}$	$\sqrt{5+2\sqrt{5}}+\sqrt{3}$	$\sqrt{15-6\sqrt{5}}-\sqrt{5}+2$	$\frac{1}{2}\left(3\sqrt{3}-\sqrt{15}-\sqrt{50-22\sqrt{5}}\right)$
$81^\circ = \frac{19\pi}{20}$	$\frac{1}{2}\left(3\sqrt{2}+\sqrt{10}+2\sqrt{5}+\sqrt{5}\right)$	$\frac{1}{2}\left(3\sqrt{2}+\sqrt{10}-2\sqrt{5+\sqrt{5}}\right)$	$\sqrt{5}+1-\sqrt{5+2\sqrt{5}}$
$84^\circ = \frac{7\pi}{15}$	$\sqrt{15+6\sqrt{5}}+\sqrt{5}+2$	$\sqrt{3}-\sqrt{5-2\sqrt{5}}$	$\frac{1}{2}\left(\sqrt{10-2\sqrt{5}}-\sqrt{15}+\sqrt{3}\right)$
$87^\circ = \frac{29\pi}{60}$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(2+\sqrt{3}+\sqrt{5+2\sqrt{5}}\right)$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(\sqrt{5+2\sqrt{5}}-2+\sqrt{3}\right)$	$\frac{1}{4}\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{3}-1\right)\left(\sqrt{10+2\sqrt{5}}-\sqrt{5}-1\right)$
$90^\circ = \frac{\pi}{2}$	∞	1	0